

HVS Mathematics Department  
**Higher GCSE Course**

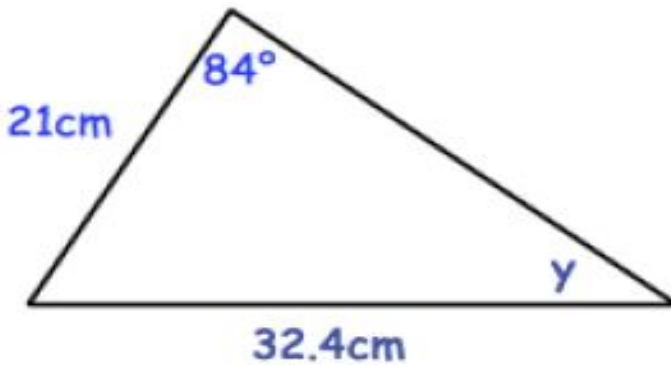
# Advanced Topics

## Year 11 Easter Booklet

# Solutions

Name .....

## Topic 1: Advanced Trigonometry



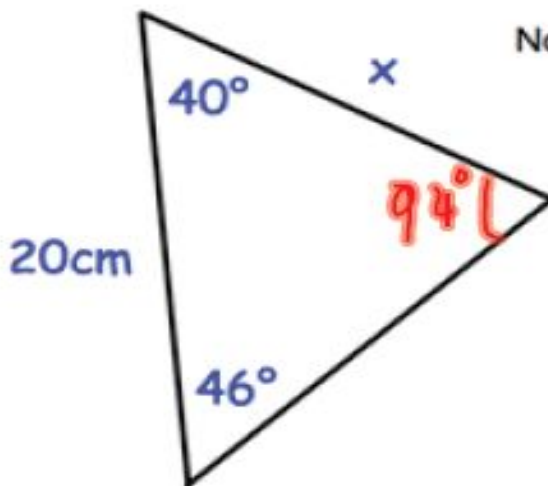
Calculate the size of the angle labelled  $y$ .

$$\frac{\sin y}{21} = \frac{\sin 84}{32.4}$$

$$\sin y = 0.644 \dots$$

$$40.14 \dots^\circ$$

(3)



Not drawn is scale.

$$180 - 40 - 46 = 94$$

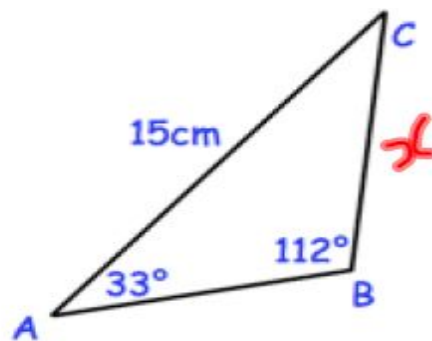
$$94^\circ$$

Find the size of  $x$ .

$$\frac{x}{\sin 46} = \frac{20}{\sin 94}$$

$$14.42 \dots \text{cm}$$

(3)



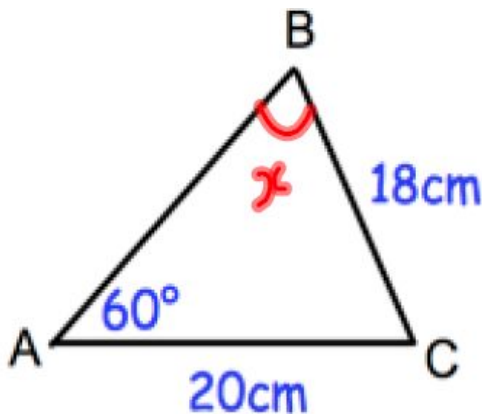
In triangle ABC the length of AC is 15cm.  
 Angle ABC = 112°  
 Angle BAC = 33°

Work out the length of BC.

$$\frac{x}{\sin 33} = \frac{15}{\sin 112}$$

$$8.81 \dots \text{cm}$$

(3)



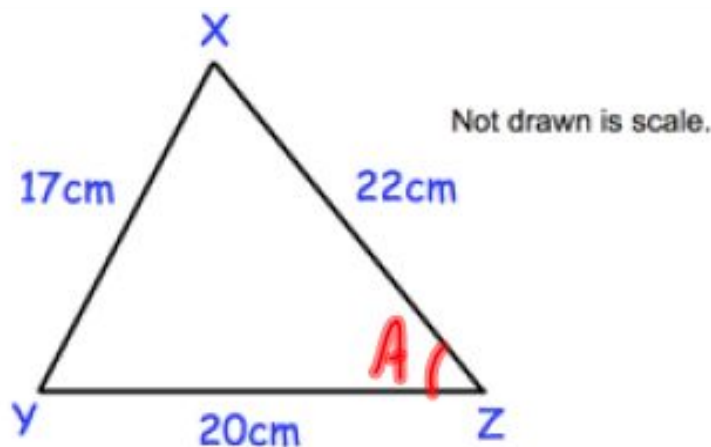
Calculate the size of angle ABC.

$$\frac{\sin x}{20} = \frac{\sin 60}{18}$$

$$\sin x = 0.962 \dots$$

$$74.2^\circ$$

(3)



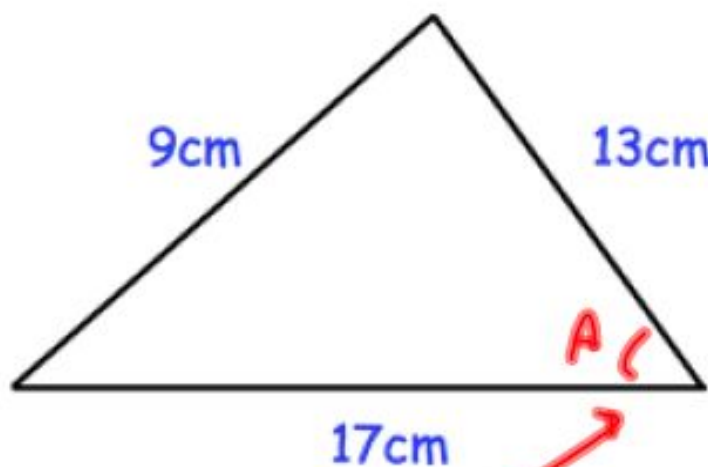
Find the size of angle XZY.

$$\cos A = \frac{20^2 + 22^2 - 17^2}{2 \times 20 \times 22}$$

$$\cos A = 0.676$$

$$A = 47.46$$

.....°  
(3)

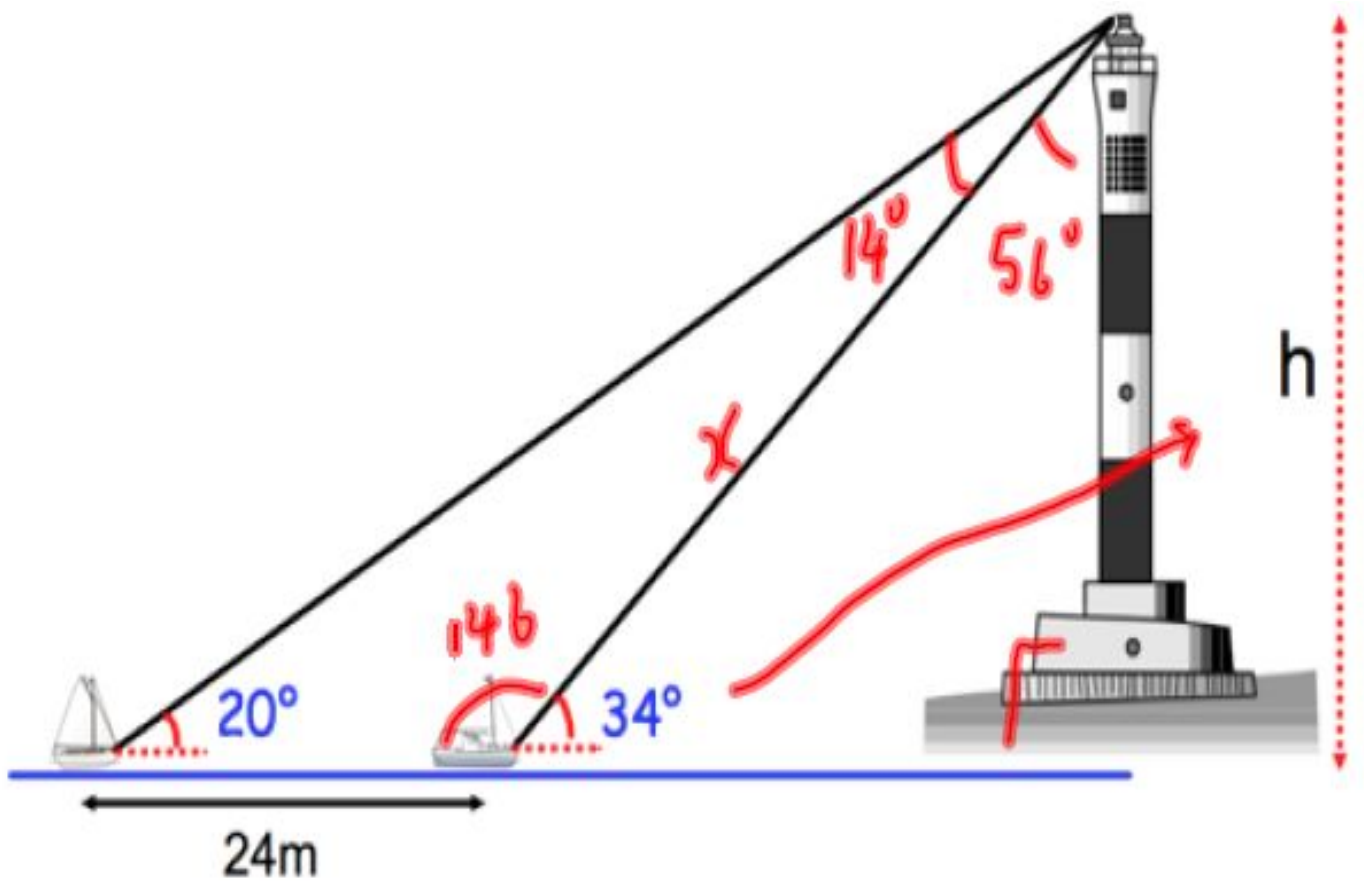


Calculate the smallest angle in the triangle.

opposite smallest side

$$\cos A = \frac{13^2 + 17^2 - 9^2}{2 \times 13 \times 17} = 0.8529...$$

31.47°  
.....°  
(3)



Two small boats are 24m apart.

The angle of elevation of the boats to the top of a lighthouse are  $20^\circ$  and  $34^\circ$  respectively.

Calculate the height of the lighthouse.

$$\frac{x}{\sin 20} = \frac{24}{\sin 14}$$

$$x = 33.93$$

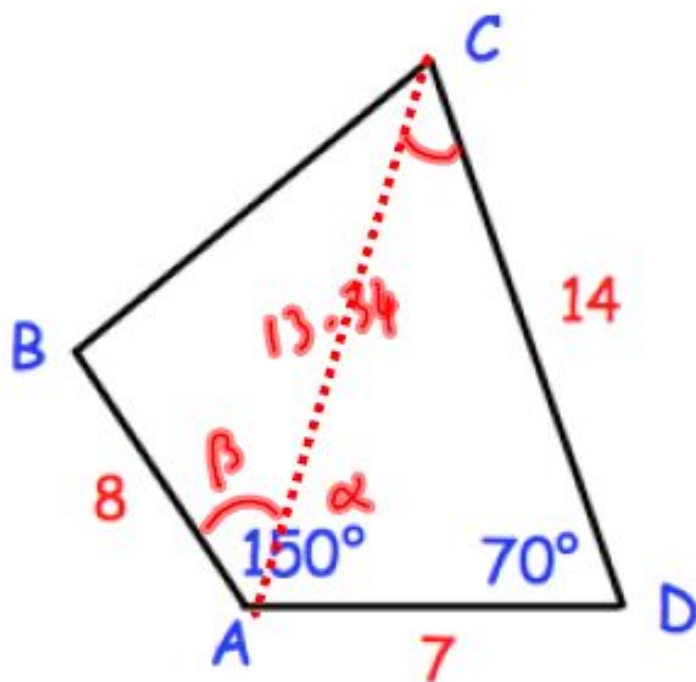
$$\sin 34 = \frac{h}{33.93}$$

$$h = 33.93 \sin 34$$

$$h = 18.97$$



In a quadrilateral ABCD,  $AD = 7\text{cm}$ ,  $AB = 8\text{cm}$  and  $CD = 14\text{cm}$ .  
 Angle  $BAD = 150^\circ$  and Angle  $ADC = 70^\circ$



Calculate the length BC.

$$AC^2 = 7^2 + 14^2 - 2 \times 7 \times 14 \times \cos 70$$

$$AC^2 = 177.91 \dots$$

$$AC = 13.34$$

$$\frac{\sin \alpha}{14} = \frac{\sin 70}{13.34}$$

$$\alpha = 80.46$$

$$\beta = 150 - 80.46 = 69.54$$

$$BC^2 = 8^2 + 13.34^2 - 2 \times 8 \times 13.34 \times \cos 69.54$$

$$BC = 12.94$$

$$12.94$$

.....cm  
 (6)

## Topic 2: Reverse Mean

MathsWatch Videos: 130

7. Shown below are five cards which are arranged in order from smallest to largest



The range of the cards is 4.

The median of the cards is 8. *middle*

The mean of the cards is 7. *— adds up to  $5 \times 7 = 35$*

Work out the 4 missing numbers.

$$5 + 8 + 9 = 22$$

$$35 - 22 = 13$$

*5   8   8   and   9*

8. 8 boys and 8 girls from a class run 100m.



The times taken, to the nearest second, for each girl are:

15    20    24    18    19    21    26    29

The mean of the boys' times is 25 seconds.

The range of the boys' times is 14 seconds.

Thomas says that "the boys in our class are faster than the girls."

Is he correct?

girls mean: 21.5 seconds

girls range: 14 seconds

No. The mean time for the girls to run 100m is 21.5 seconds, which is less than the mean time for the boys. Therefore on average the girls were faster. The ranges for both the boys and girls are equal which suggests there are no outliers that are affecting the mean.



7.

Simon rolls a fair six-sided dice 30 times.

He records the results in a table, however misses two of the frequencies.

Number	Frequency
1	6
2	3
3	5
4	8
5	2
6	6

 $f \times$ 

6

6

15

 $+ 36$ 105

30

} 42

The mean result is 3.5

Work out the two missing numbers.

$$3.5 \times 30 = 105$$

$$6 + 6 + 15 + 36 = 63$$

$$105 - 63 = 42$$

$$4 \times 8 = 32$$

$$5 \times 2 = 10$$

$$6 + 3 + 5 + 8 + 2 + 6 = \underline{\underline{30}}$$

8. The star rating, from 1 to 5, of hotels in a city are summarised below.



Star rating $x$	Frequency $f$
1	4
2	17
3	23
4	$x$
5	18

$$+ \frac{18}{62+x}$$

$$\begin{array}{r} f \cdot x \\ 4 \\ 34 \\ 69 \\ 4x \\ + 90 \\ \hline 197 + 4x \end{array}$$

The mean star rating of a hotel in the city is 3.575

Calculate the value of the missing frequency,  $x$ .

$$\frac{197 + 4x}{62 + x} = 3.575$$

$$197 + 4x = 3.575(62 + x)$$

$$197 + 4x = 221.65 + 3.575x$$

$$0.425x = 24.65$$

$$x = 58$$

58

### Topic 3: Equations of Lines Advanced

13. The point A (-3, 5) and the point B (1, -15) lie on the line L.

Find the equation of the line L.

$$m = \frac{-15 - 5}{1 - -3} = \frac{-20}{4} = -5$$

$$(1, -15) \rightarrow y = -5x + c$$

$$-15 = -5 \times 1 + c$$

$$-15 = -5 + c$$

$$c = -10$$

$$y = -5x - 10$$

.....

(4)

- 
14. The point A (1, 1) and the point B (5, -1) lie on the line L.

Find the equation of the line L.

$$m = \frac{-1 - 1}{5 - 1} = \frac{-2}{4} = -\frac{1}{2}$$

$$(1, 1) \rightarrow y = -\frac{1}{2}x + c$$

$$1 = -\frac{1}{2} \times 1 + c$$

$$1 = -\frac{1}{2} + c$$

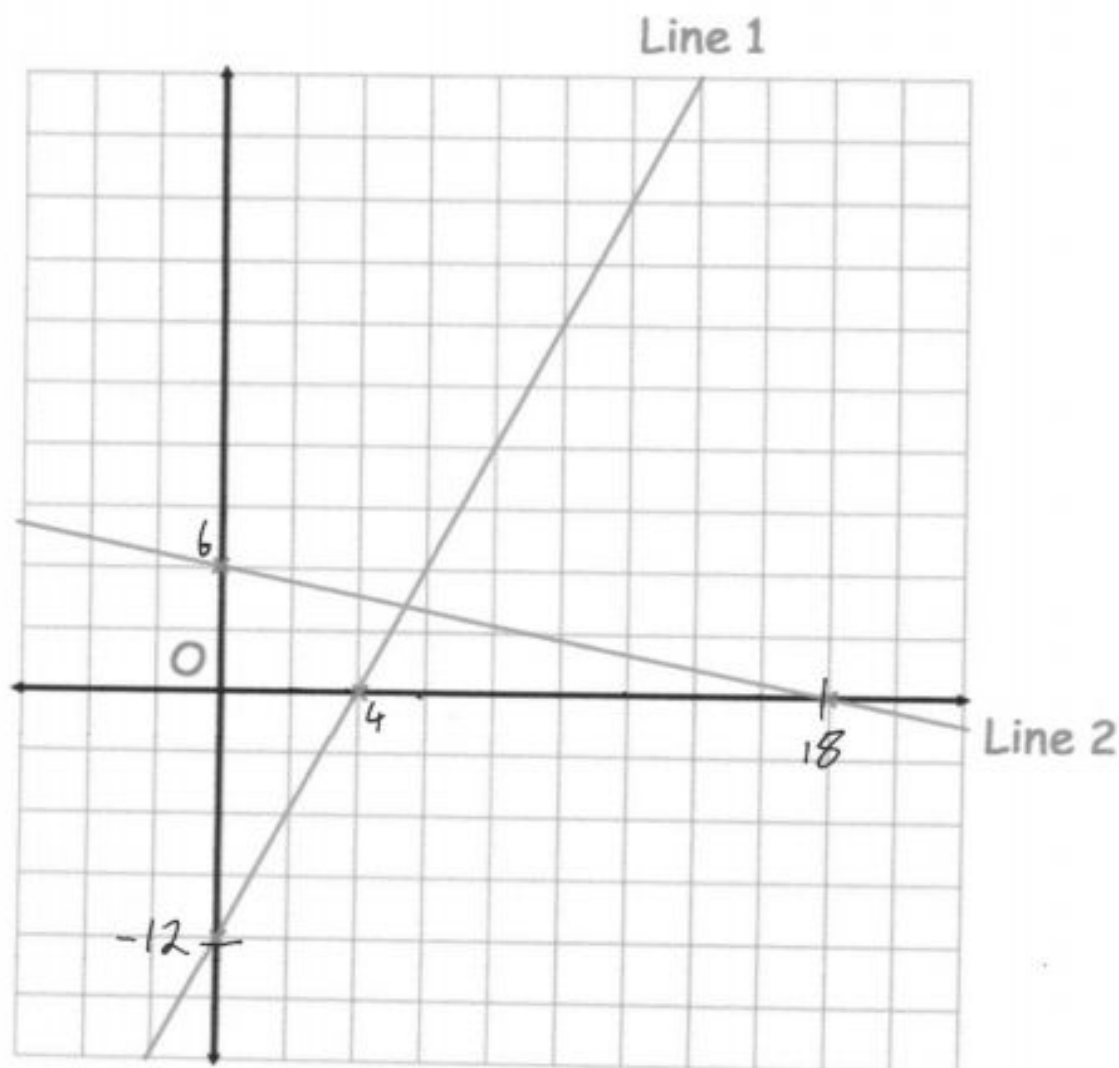
$$c = 1.5$$

$$y = -\frac{1}{2}x + 1.5$$

.....

(4)

Shown are two straight lines drawn on the grid.



Line 1 has equation  $y = 3x - 12$

(a) Find the equation of Line 2

$$0 = 3x - 12$$

$$x = 4$$

$$y = -\frac{1}{3}x + 6$$

(4)

(b) Are the two lines perpendicular?  
Explain your answer.

yes

$$m_1 \times m_2 = -1 \text{ if perpendicular}$$

$$3 \times -\frac{1}{3} = -1 \quad \checkmark$$

(1)

5. The line  $l$  is a tangent to the circle  $x^2 + y^2 = 68$  at the point  $P$ .  
 $P$  is the point  $(2, 8)$

Work out the equation of the line  $l$

gradient of  $OP = 4$

$$y = -\frac{1}{4}x + c$$

$$8 = -\frac{1}{4} \times 2 + c$$

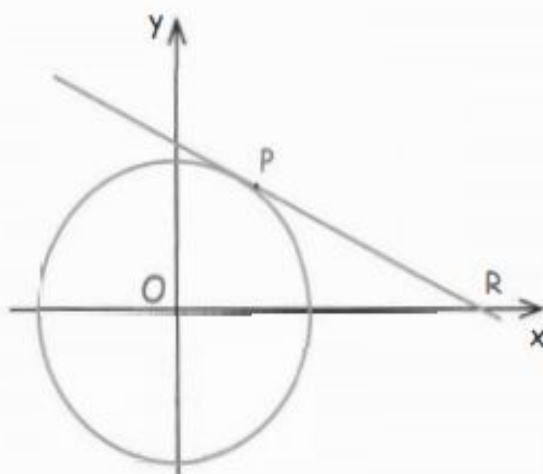
$$c = 8\frac{1}{2} \text{ or } 8.5$$

$$y = -0.25x + 8.5$$

$$y = -\frac{1}{4}x + \frac{17}{2}$$

(3)

7. The diagram shows the circle  $x^2 + y^2 = 17$



$$x^2 + y^2 = 17$$

$$y = \pm 4$$

$P$  lies on the circle and has  $x$ -coordinate 1.  
 The tangent at  $P$  intersects the  $x$ -axis at  $R$ .

$$(1, 4)$$

Work out the coordinates of  $R$

gradient of  $OP = 4$

$$y = -\frac{1}{4}x + c$$

$$4 = -\frac{1}{4} \times 1 + c$$

$$c = 4\frac{1}{4}$$

$$c = \frac{17}{4}$$

$$y = -\frac{1}{4}x + \frac{17}{4}$$

$$y = -0.25x + 4.25$$

when  $y = 0$

$$R = (17, 0)$$

(5)



The line  $l$  is a tangent to the circle  $x^2 + y^2 = 90$  at the point P.

P is the point (3, 9)

The line  $l$  crosses the x-axis at the point Q.

Work out the area of triangle OPQ.

gradient of OP = 3

$$y = -\frac{1}{3}x + c$$

$$9 = -1 + c$$

$$c = 10$$

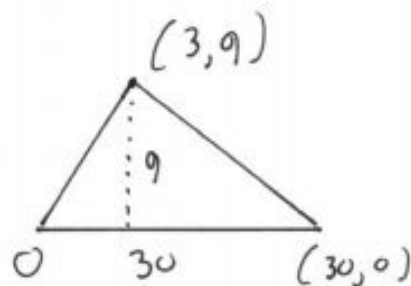
$$y = -\frac{1}{3}x + 10$$

Q

$$0 = -\frac{1}{3}x + 10$$

$$x = 30$$

$$Q(30, 0)$$



$$\frac{1}{2} \times 30 \times 9$$

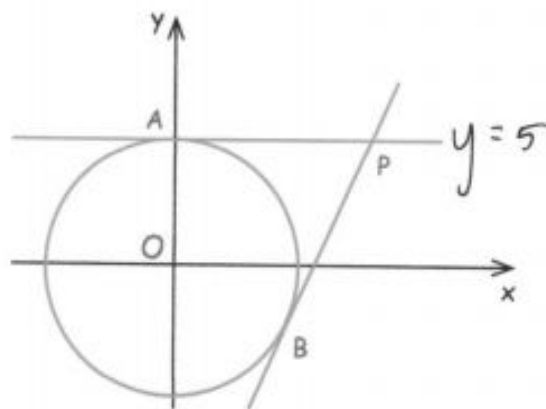
$$135$$

(5)

The circle  $x^2 + y^2 = 25$  has tangents at the points A and B.

The point A has coordinates (0, 5)

The point B has coordinates (3, -4)



The tangents meet at the point P.

Work out the coordinates of the point P.

gradient of OB =  $-\frac{4}{3}$

$$y = \frac{3}{4}x + c$$

$$-4 = \frac{3}{4} \cdot 3 + c$$

$$c = -\frac{25}{4}$$

$$y = \frac{3}{4}x - 6.25$$

$$\frac{3}{4}x - 6.25 = 5$$

$$\frac{3}{4}x = 11.25$$

$$x = 15$$

$$(15, 5)$$

(5)

## Topic 4: Upper and Lower Bounds

14. Sophie estimated that the distance between Bristol and Newcastle is about 290 miles and that her average driving speed would be 60 mph.



She estimated the distance to the nearest 10 miles and the speed to the nearest 10 mph.

*quickest time*

Calculate the lower bound of the time the journey should take.

Give your answer in hours and minutes.

Give your answer to the nearest minute.

*quickest* →  $t = \frac{d}{s}$  *min* ← *max*

$$t = \frac{285}{65} = 4.384615$$

$$0.384615 \times 60 = 23.076...$$

4 ..... hours 23 ..... minutes  
(4)

15. The final velocity of a traveling object is given by the formula



$$v = u + at$$

where  $v$  is the final velocity

$u$  is the initial velocity

$a$  is the acceleration

and  $t$  is the time

Given  $u = 2.4$  m/s correct to 2 significant figures,  $2.35$   
 $a = 12$  m/s<sup>2</sup> correct to 2 significant figures,  $11.5$   
and  $t = 5$  seconds correct to 1 significant figure,  $4.5$

Calculate the lower bound for  $v$ .

$$v = 2.35 + 11.5 \times 4.5$$

=

54.1 ..... m/s  
(4)

16. The curved surface area of a cone is given by the formula



$$A = \pi r l$$

$\uparrow$   $\uparrow$   
max min

where  $A$  is the curved surface area  
 $r$  is the radius of the base of the cone  
and  $l$  is the slant height

Given  $A = 220 \text{ cm}^2$  correct to 3 significant figures,  
and  $r = 8 \text{ cm}$  correct to 1 significant figure.

Calculate the upper bound for  $l$ .

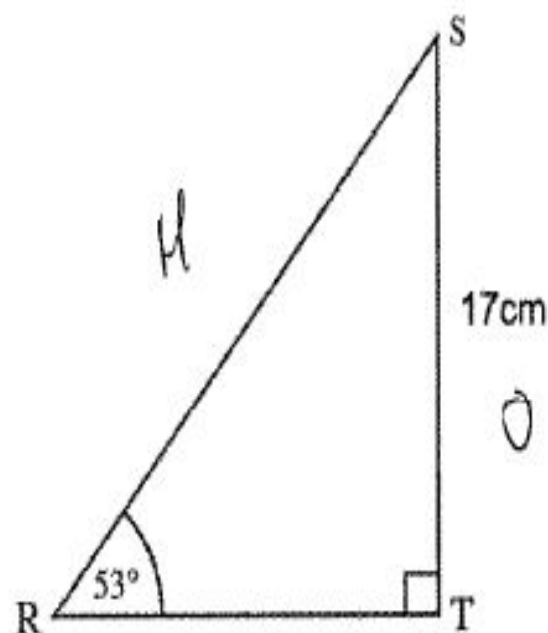
$$220.5 = \pi \times 7.5 \times l$$

$$\frac{220.5}{7.5\pi} = l$$

$$9.3583\dots \text{cm}$$

(3)

17. Shown below is triangle RST.  
 Angle SRT is  $53^\circ$ , to the nearest degree.  
 ST is 17cm to the nearest centimetre.



Work out the upper bound for the length of RS.

$5^\circ H$

$$RS = \frac{0 \leftarrow \text{max}}{\sin 2 \leftarrow \text{min}}$$

$$RS = \frac{17.5}{\sin 52.5} =$$

22.058 cm  
 (4)

18. The length of the base of a triangle is 10cm, correct to 2 significant figures.  
The length of the perpendicular height of a triangle is 15cm, correct to 2 significant figures.



(a) Calculate the upper bound for the area of the triangle

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 10.5 \times 15.5$$

=

$$\begin{array}{r} 81.375 \\ \hline \text{.....cm}^2 \\ (2) \end{array}$$

(b) Calculate the lower bound for the area of the triangle

$$A = \frac{1}{2} \times 9.95 \times 14.5$$

=

$$\begin{array}{r} 72.1375 \\ \hline \text{.....cm}^2 \\ (2) \end{array}$$

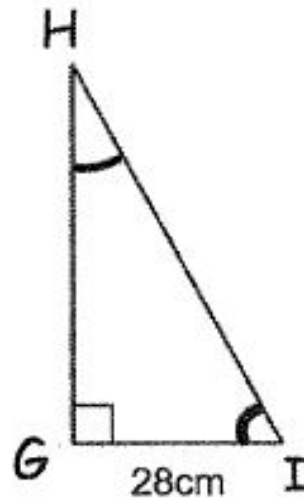
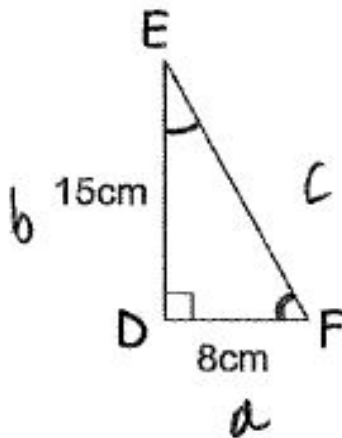


## Topic 5: Similarity – Area and Volume

11.



Not drawn to scale



DEF and GHI are similar right angled triangles.

DE = 15cm

DF = 8cm

GI = 28cm

$$\begin{aligned}\text{Scale factor} &= 28 \div 8 \\ &= 3.5\end{aligned}$$

Work out the length of HI

$$EF^2 = DE^2 + DF^2$$

$$EF^2 = 15^2 + 8^2$$

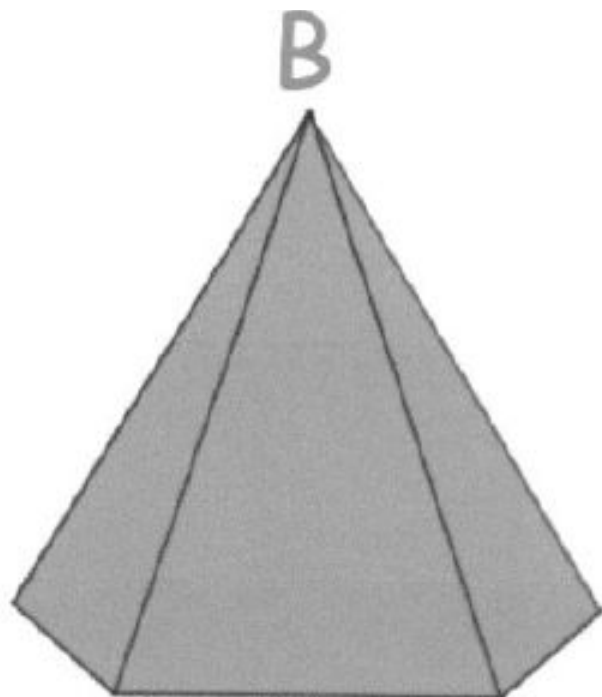
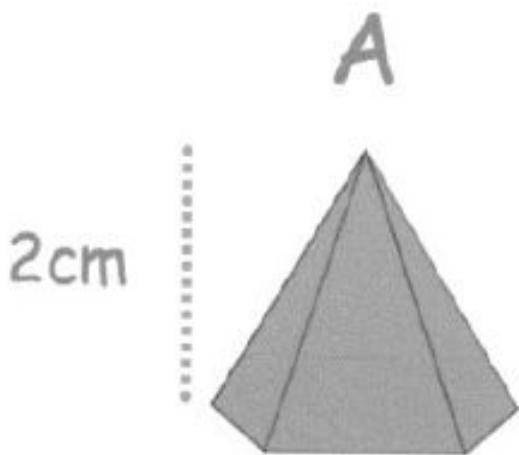
$$EF^2 = 289$$

$$EF = 17$$

$$HI = 17 \times 3.5$$

59.5 cm

(5)



Pyramid A has a surface area of  $20\text{cm}^2$   
 Pyramid B has a surface area of  $320\text{cm}^2$   
 The height of pyramid A is  $2\text{cm}$

(a) Work out the height of pyramid B.

$$320 \div 20 = 16$$

$$\sqrt{16} = 4$$

$$2 \times 4$$

8 cm  
 (3)

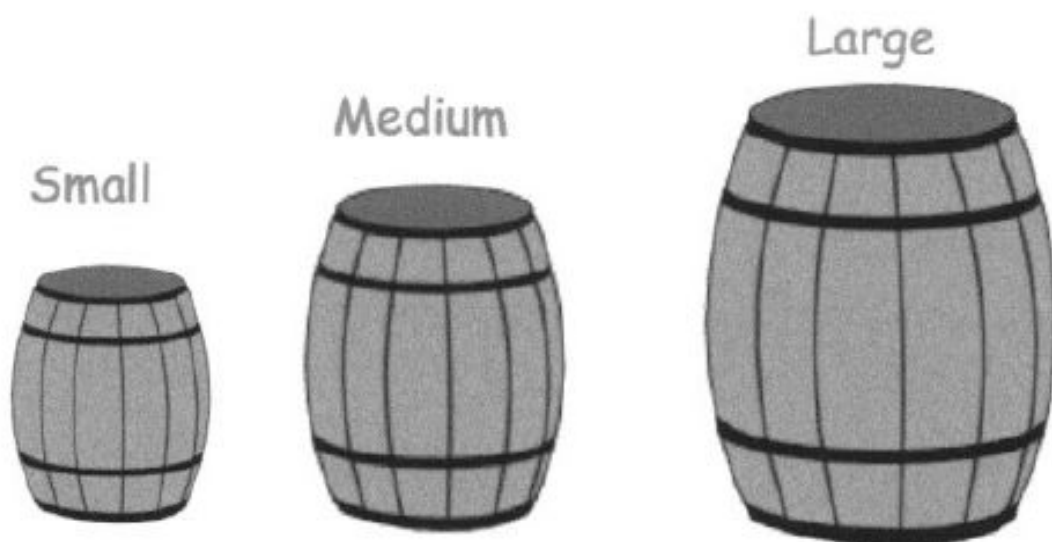
Pyramid A has a weight of  $800\text{g}$   
 Both pyramids are made of the same material.

(b) Work out the weight of pyramid B.  
 Include suitable units.

$$0.8\text{kg} \times 4^3$$

51.2kg  
 (4)

15. Wine is kept in three different sized barrels that are mathematically similar.



The table below shows information about the barrels.


	Height	Capacity
Small	24 inches	100 litres
Medium	30 inches	195.3125 litres
Large	32.1327816 inches	240 litres

Complete the table.

$$\frac{30}{24} = 1.25 \quad 100 \times 1.25^3$$

$$240 \div 100 = 2.4 \quad \sqrt[3]{2.4} = 1.3388659..$$

$$24 \times 1.3388.. = 32.132$$

21.  Two solids are mathematically similar.  
The surface area of the smaller solid is  $42\pi \text{ cm}^2$   
The surface area of the larger solid is  $1512\pi \text{ cm}^2$

The height of the larger solid is 96cm.


Work out the height of the smaller solid.

$$1512\pi \div 42\pi = 36$$

$$\sqrt{36} = 6$$

$$96 \div 6 =$$

16 .....cm  
(3)

22.  The areas of two mathematically similar shapes are in the ratio 49 : 81  
The length of the smaller shape is 24.5cm


Work out the length of the larger shape.

$$7 : 9$$

$$24.5 \div 7 = 3.5$$

$$3.5 \times 9$$

31.5 .....cm  
(3)

23.  The volumes of two mathematically similar solids are in the ratio 8 : 125  
The surface area of the smaller solid is  $24 \text{ cm}^2$

Work out the surface area of the larger solid.

sides  $2 : 5$

Areas  $4 : 25$

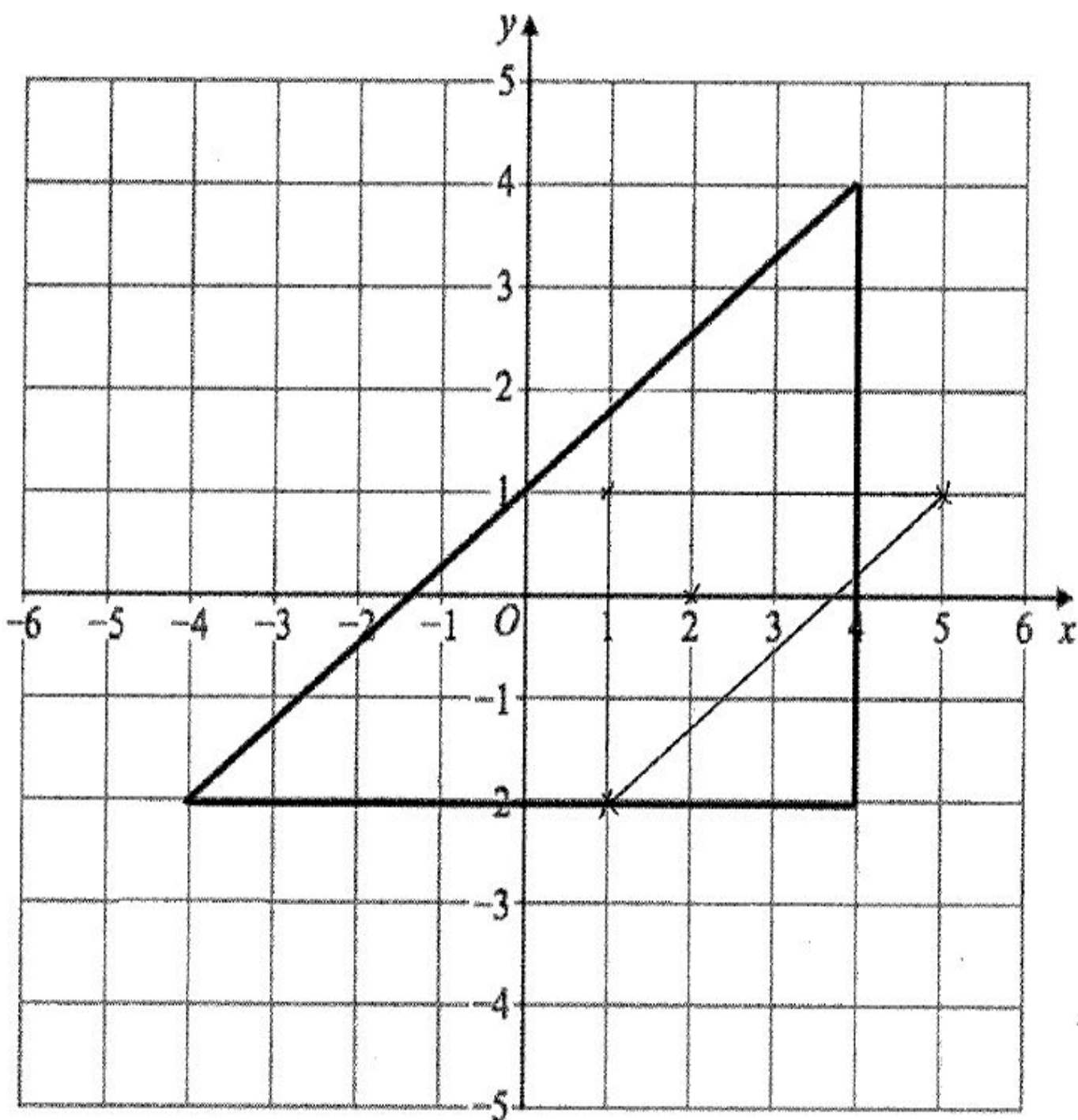
$$24 \div 4 = 6$$

$$6 \times 25$$

150 .....cm<sup>2</sup>  
(3)

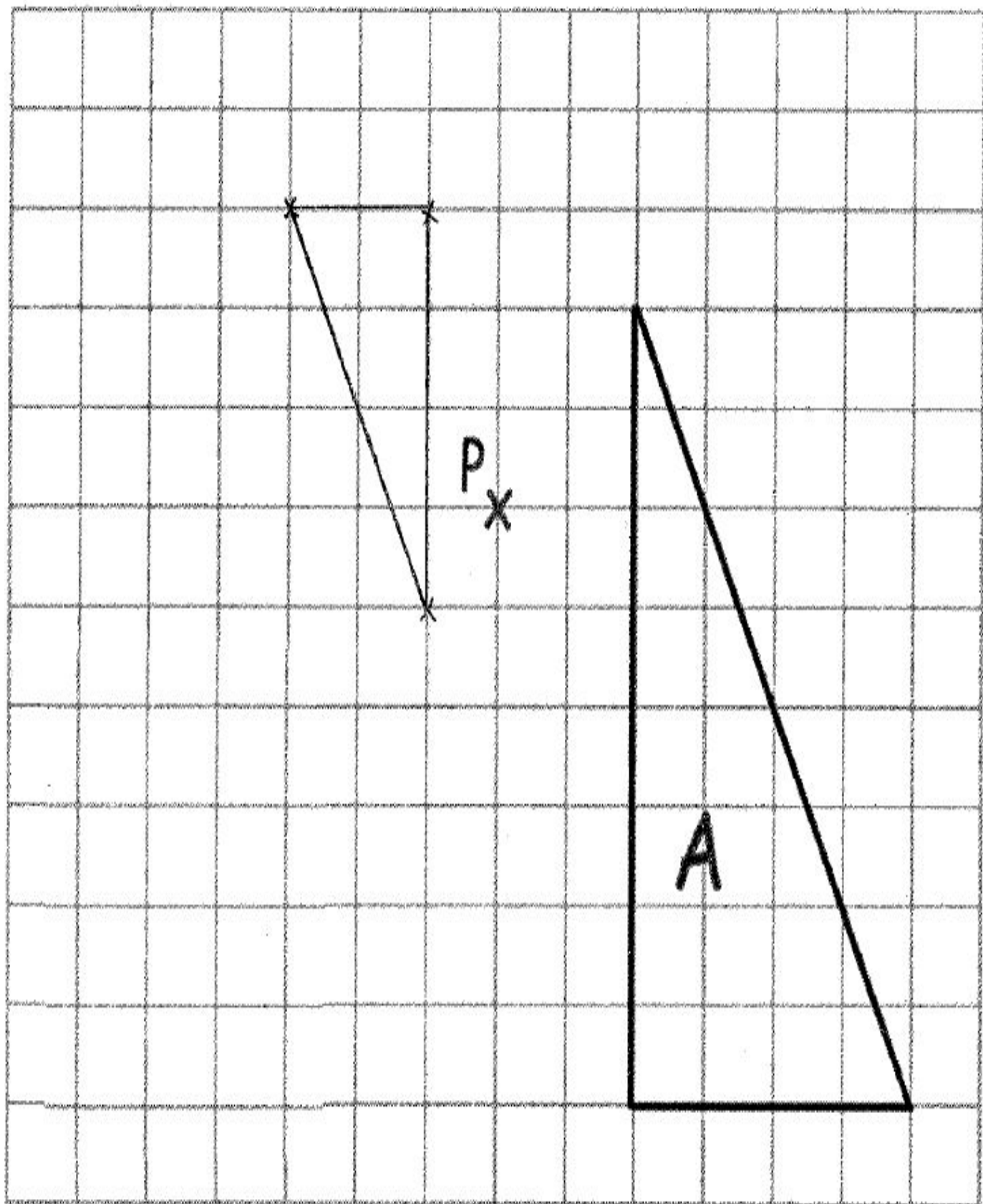
## Topic 6: Enlargement – Negative Fractional

5.



Enlarge the triangle by scale factor  $-\frac{1}{2}$ , using centre of enlargement  $(2, 0)$





Enlarge shape A by scale factor  $-\frac{1}{2}$ , using the point P as centre of enlargement.

## Topic 7: Composite & Inverse Functions

3. The functions  $f(x)$ ,  $g(x)$  and  $h(x)$  are given by the following:

$$f(x) = x^2 - 3$$

$$g(x) = 2x + 1$$

$$h(x) = \frac{x}{2}$$

- (a) Find  $fg(x)$

$$\begin{aligned} &(2x+1)^2 - 3 \\ &(2x+1)(2x+1) - 3 \\ &= 4x^2 + 2x + 2x + 1 - 3 \end{aligned}$$

$$fg(x) = 4x^2 + 4x - 2 \quad (2)$$

- (b) Find  $gh(x)$

$$2\left(\frac{x}{2}\right) + 1 = x + 1$$

$$gh(x) = x + 1 \quad (2)$$

- (c) Find  $h^{-1}(x)$

$$y = \frac{x}{2}$$

$$2y = x$$

$$h^{-1}(x) = 2x \quad (2)$$

4. The function  $f$  is such that  $f(x) = 4x - 7$

- (a) Solve  $f(x) = 17$

$$\begin{aligned} 4x - 7 &= 17 \\ +7 &+7 \\ 4x &= 24 \\ x &= 6 \end{aligned}$$

$$x = 6 \quad (2)$$

- (b) Find  $f^{-1}(x)$

$$\begin{aligned} y &= 4x - 7 \\ y + 7 &= 4x \\ \frac{y+7}{4} &= x \end{aligned}$$

$$f^{-1}(x) = \frac{x+7}{4} \quad (2)$$

5. Given  $f(x) = x^2 + 2$  and  $g(x) = x + 14$

Find the values of  $a$  such that  $f(a) = g(a)$

$$a^2 + 2 = a + 14$$

$$a^2 - a - 12 = 0$$

$$(a-4)(a+3) = 0$$

$$a = 4 \text{ or } -3$$

(3)

6. The functions  $f(x)$  and  $g(x)$  are given by the following:

$$f(x) = 8 - 3x$$

$$g(x) = 4x$$

- (a) Calculate the value of  $gf(3)$

$$f(3) = 8 - (3 \times 3) = -1$$

$$g(-1) = 4 \times -1$$

$$-4$$

(2)

- (b) Solve the equation  $gf(x) = 80$

$$gf(x) = 4(8 - 3x) = 32 - 12x$$

$$32 - 12x = 80$$

$$-48 = 12x$$

$$x = -4$$

(4)

7.  $f(x) = \frac{3x}{5} + 1$        $y = \frac{3x}{5} + 1$

Find  $f^{-1}(x)$

$$y = \frac{3x}{5} + 1$$

$$5y - 5 = 3x$$

$$\frac{5y - 5}{3} = x$$

$$f^{-1}(x) = \frac{5x - 5}{3}$$

(3)

8. Given  $f(x) = x^2 + 3x - 5$

Express  $f(2x - 1)$  in the form  $ax^2 + bx + c$

$$\begin{aligned} f(2x-1) &= (2x-1)^2 + 3(2x-1) - 5 \\ &= (2x-1)(2x-1) + 6x - 3 - 5 \\ &= 4x^2 - 4x + 1 + 6x - 8 \\ &= 4x^2 + 2x - 7 \end{aligned}$$

$$\underline{4x^2 + 2x - 7}$$

(3)

9. The function  $f$  is such that  $f(x) = kx + 3$

The function  $g$  is such that  $g(x) = 2x - 4$

Given that  $gf(2) = 34$

work out the value of  $k$

$$\begin{aligned} gf(x) &= 2(kx + 3) - 4 = 2kx + 6 - 4 \\ &= 2kx + 2 \end{aligned}$$

$$\begin{aligned} gf(2) &= 4k + 2 = 34 \\ k &= 8 \end{aligned}$$

$$\underline{k = 8}$$

(3)

10. For all values of  $x$ ,

$$f(x) = x^2 + 4$$

$$g(x) = x - 9$$

$$\begin{aligned} gf(x) &= (x^2 + 4) - 9 \\ &= x^2 - 5 \end{aligned}$$

Solve  $fg(x) = gf(x)$

$$fg(x) = gf(x)$$

$$\begin{aligned} fg(x) &= (x-9)^2 + 4 \\ &= (x-9)(x-9) + 4 \\ &= x^2 - 18x + 81 + 4 \\ &= x^2 - 18x + 85 \end{aligned}$$

$$\begin{aligned} x^2 - 18x + 85 &= x^2 - 5 \\ -18x + 85 &= -5 \\ -18x &= -90 \end{aligned}$$

$$\underline{x = 5}$$

(4)

11.  $f(x) = x^2 + 2x + 1$

Show that  $f(x+2) - f(x) = 4x + 8$

$$f(x) = x^2 + 2x + 1$$

$$\begin{aligned} f(x+2) &= (x+2)^2 + 2(x+2) + 1 \\ &= x^2 + 4x + 4 + 2x + 4 + 1 \\ &= x^2 + 6x + 9 \end{aligned}$$

$$f(x+2) - f(x) = 4x + 8$$

QED



## Topic 8: Compounds Units

9. The mass of  $3\text{m}^3$  of tin is  $21840\text{kg}$ .



- (a) Work out the density of tin.

$$\frac{m}{v} = \frac{21840\text{ kg}}{3\text{ m}^3}$$

$$= 7280 \text{ kg/m}^3$$

(2)

The density of aluminium is  $2712\text{kg/m}^3$ .  $\times 5 = 13560\text{ kg}$

- (b) Work out the difference in mass between  $5\text{m}^3$  of tin and  $5\text{m}^3$  of aluminium.

$$\text{tin} - 21840\text{ kg} \div 3 \times 5 = 36400\text{ kg}$$

$$\text{Aluminium} - 13560\text{ kg}$$

$$= 22840 \text{ kg}$$

(3)

10. Mr. Dixon is building a toy boat for his son.  
He has three different planks of wood to choose from.



Plank A	Plank B	Plank C
Volume = $750\text{cm}^3$ Mass = $900\text{g}$	Volume = $0.0152\text{m}^3$ Mass = $7.6\text{kg}$	Volume = $1000\text{cm}^3$ Mass = $1.02\text{kg}$

If wood has a density under  $1\text{g/cm}^3$ , it will float.

Which plank of wood is the most suitable?  
Explain your answer.

$$D = \frac{m}{v}$$

$$\frac{900}{750} = 1.2\text{ g/cm}^3$$

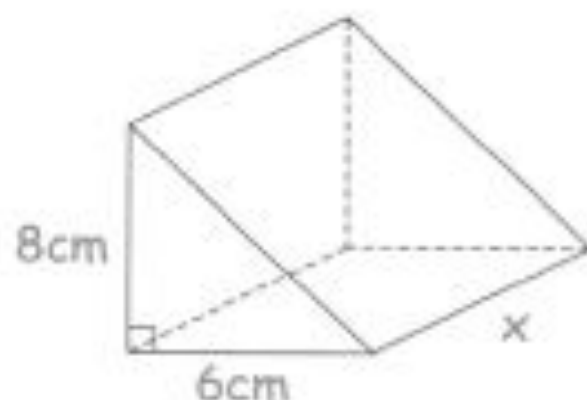
$$\frac{7600}{15200} = 0.5\text{ g/cm}^3$$

$$\frac{1020}{1000} = 1.02\text{ g/cm}^3$$

Plank B

(4)

13. The diagram shows a solid triangular prism.



The prism is made from wood and has a mass of 643.8g  
The density of wood is  $1.85 \text{ g/cm}^3$

Calculate the length of the prism.

$$\frac{M}{D} = \frac{643.8}{1.85} = 348 \text{ cm}^3$$

$$24 \times x = 348$$

$$\frac{14.5}{\dots} \text{ cm} \quad (4)$$

14. Material A has a density of  $5.8 \text{ g/cm}^3$ .  $65 \text{ cm}^3$   
Material B has a density of  $4.1 \text{ g/cm}^3$ .  $400 \text{ cm}^3$



377g of Material A and 1.64kg of Material B form Material C.

Work out the density of Material C.

Volume of A

$$\frac{377}{5.8} = 65 \text{ cm}^3$$

Volume of B

$$\frac{1640}{4.1} = 400 \text{ cm}^3$$

$$D = \frac{M}{V}$$

$$(377 + 1640)$$

$$(65 + 400)$$

$$\frac{4.3376}{\dots} \text{ g/cm}^3 \quad (4)$$

15. The diagram shows a solid glass paperweight.



The paperweight is a hemisphere with diameter 6cm.  
The glass has a density of  $3\text{g/cm}^3$ .

Calculate the mass of the paperweight.

$$\begin{aligned} M &= D \times V \\ &= 3 \times 56.548 \dots \\ &= 169.646 \end{aligned}$$

$$\begin{aligned} V &= \frac{1}{2} \left( \frac{4}{3} \pi r^3 \right) \\ &= \frac{1}{2} \left( \frac{4}{3} \times \pi \times 3^3 \right) \\ &= 56.548 \dots \text{cm}^3 \end{aligned}$$

$$\begin{array}{r} 169.646 \\ \hline \text{g} \\ (4) \end{array}$$

16. A solid metal sphere has a radius of 4cm, correct to the nearest centimetre.  
Mass of the sphere is  $720\text{g}$ , correct to two significant figures.



Work out the greatest possible density of the metal.  
Give your answer to three significant figures.

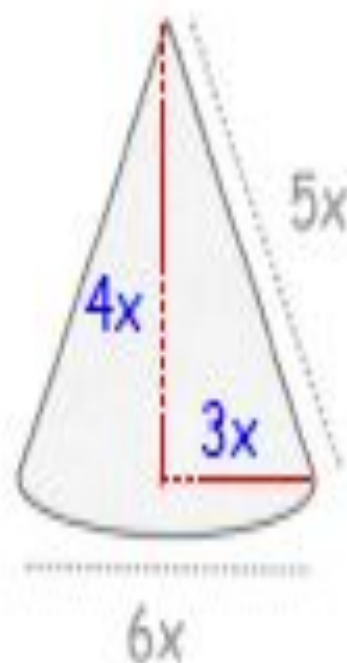
$$D = \frac{M}{V} \quad \begin{array}{l} \swarrow \uparrow \\ \searrow \downarrow \end{array} \quad \begin{array}{r} 725 \\ \hline 179.59438 \end{array}$$

$$V = \frac{4}{3} \times \pi \times 3.5^3$$

$$\frac{4}{3} \times \pi \times 3.5^3 = 179.594 \dots \text{cm}^3$$

$$\begin{array}{r} 4.04 \\ \hline \text{g/cm}^3 \\ (4) \end{array}$$

17. The diagram below shows a solid cone.



$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \times \pi \times (3x)^2 \times 4x$$

$$V = \frac{1}{3} \times \pi \times 9x^2 \times 4x$$

The cone is made from a material with density  $5\text{g/cm}^3$ .

Write an expression for the mass of the cone, in terms of  $x$ .

$$= \frac{1}{3} \times \pi \times 36x^3$$

$$= 12\pi x^3$$

$$m = D \times V$$

$$m = 5 \times 12\pi x^3$$

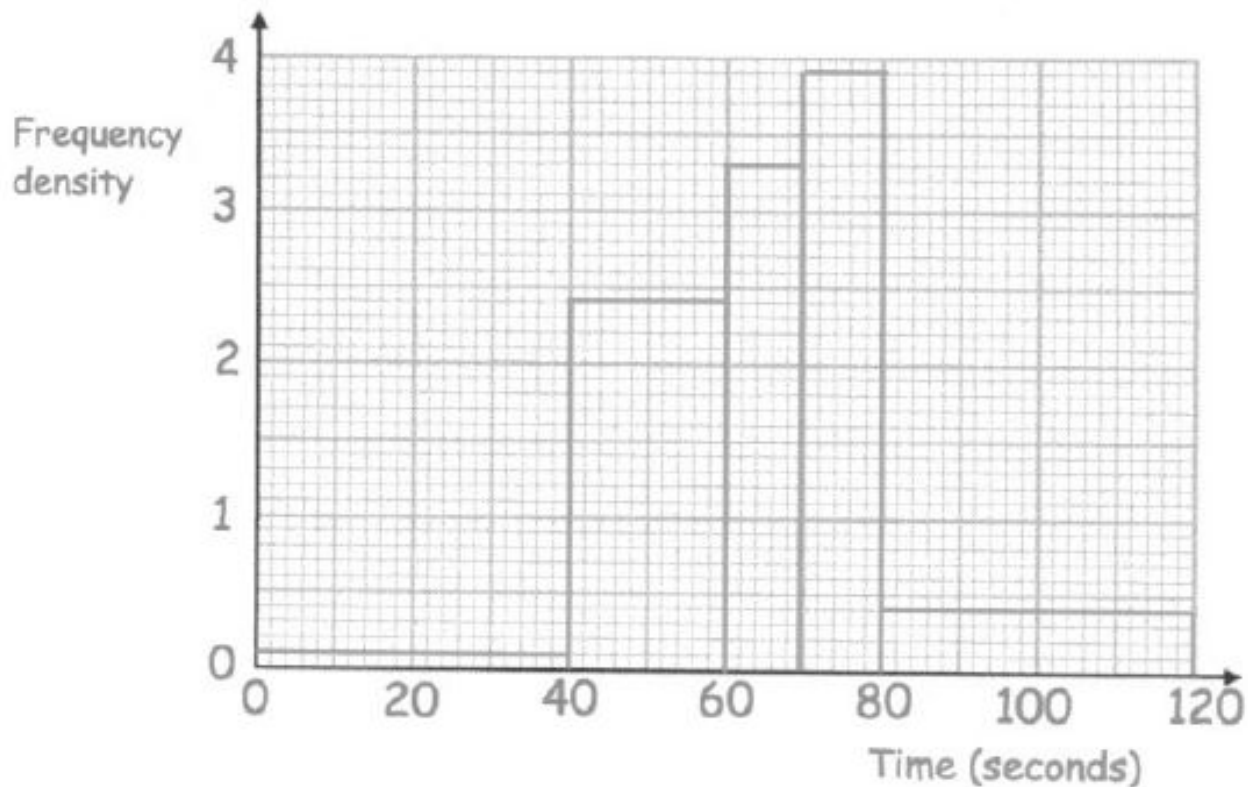
$$= 60\pi x^3$$

$$\therefore 60\pi x^3 \text{ cm}^3$$

(4)

## Topic 9: Histograms

17. The histogram shows information about the time taken by 140 students to complete a puzzle.



- (a) Complete this frequency table.

Time, $t$ seconds	Frequency
$0 < t \leq 40$	4
$40 < t \leq 60$	48
$60 < t \leq 70$	33
$70 < t \leq 80$	39
$80 < t \leq 120$	16

$$20 \times 2.4$$

$$10 \times 3.9$$

- (b) Calculate an estimate of the median.

(2)

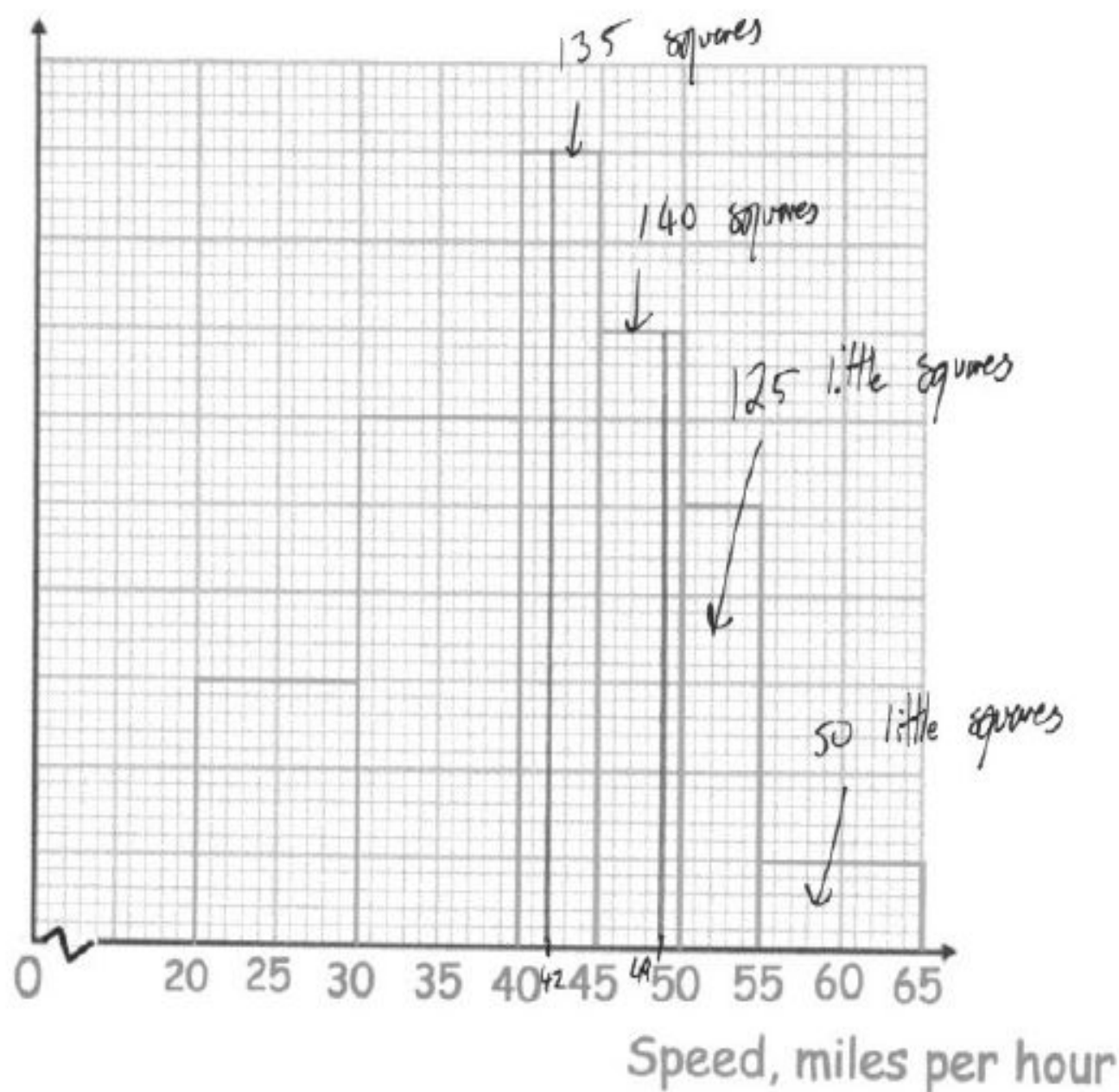
70<sup>th</sup> value.

$$60 + \frac{18}{33} \times 10 =$$

$$\underline{65.455 \text{ seconds}}$$

(3) to 3 dp

15. The histogram shows the speeds in miles per hour of 82 cars on a road.



14 cars were travelling over 50 mph.

Calculate an estimate of the number of cars that were travelling between 42 and 49 mph.

$$175 \text{ squares} = 14 \text{ cars}$$

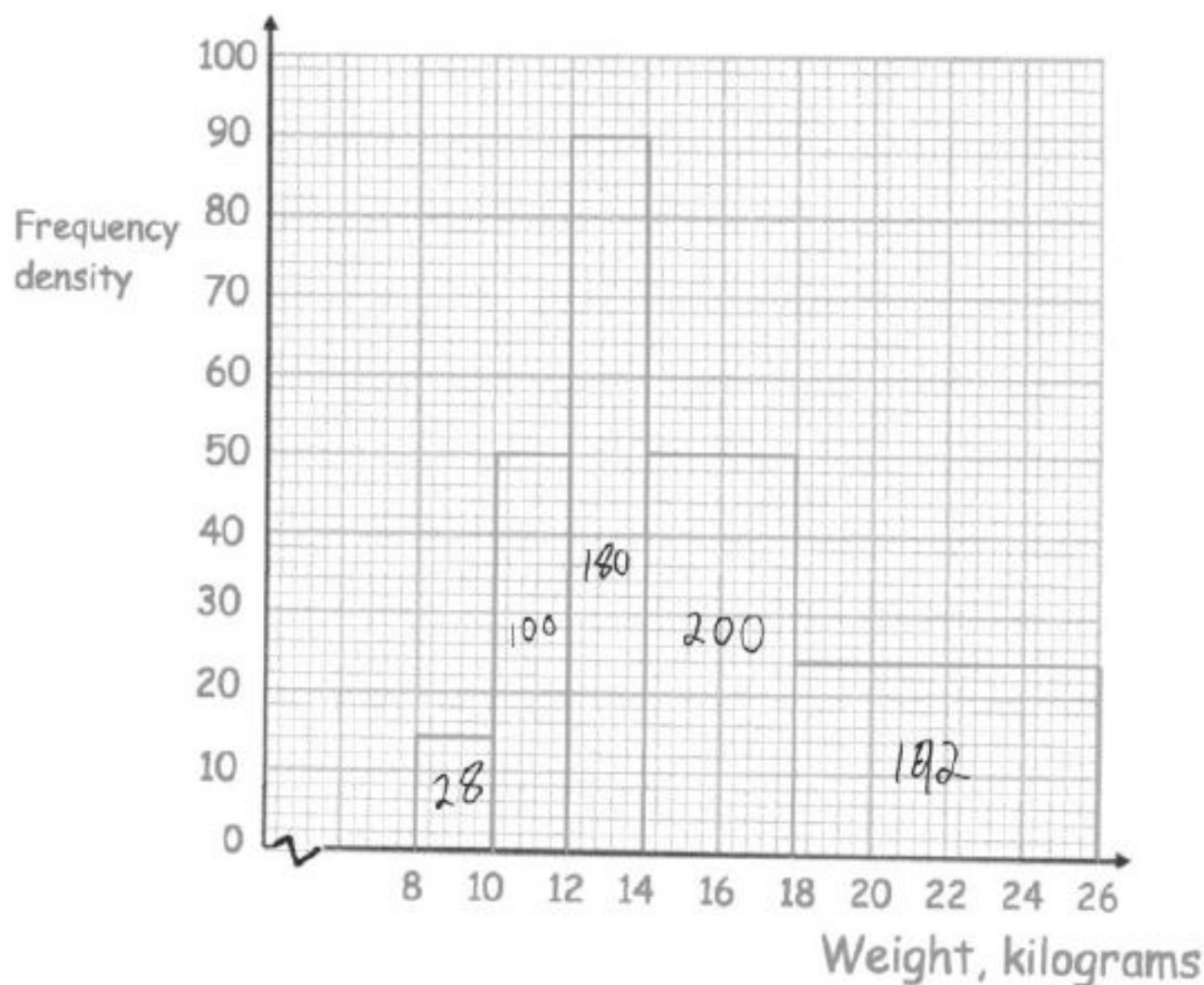
$$12.5 \text{ squares} = 1 \text{ car}$$

$$275 \div 12.5 = 22 \text{ cars}$$

22

(4)

18. The histogram shows the weights of 700 dogs.



(a) Calculate an estimate of the median.

350<sup>th</sup>

42<sup>nd</sup> int.

$14 \leq w < 18$

$$14 + \frac{42}{200} \times 4$$

14.84

(3)

(b) Calculate an estimate of the upper quartile.

525<sup>th</sup>

17<sup>th</sup> int.  $18 \leq w < 26$

$$18 + \frac{17}{192} \times 8$$

18.708

(3)



## Topic 10: Recurring Decimals to Fractions

MathsWatch Video: 177

4. Write  $0.\dot{8}1$  as a fraction.  
Give your answer in its simplest form.

$$\begin{aligned}x &= 0.81818181\dots \\100x &= 81.8181\dots \\99x &= 81 \\x &= \frac{81}{99}\end{aligned}$$

$$\frac{9}{11}$$

(3)

5. Convert  $0.3\dot{4}$  to a fraction.  
Give your answer in its simplest form.

$$\begin{aligned}x &= 0.344444\dots \\10x &= 3.44444\dots \\100x &= 34.4444\dots \\90x &= 31\end{aligned}$$

$$\frac{31}{90}$$

(3)

6. Write  $0.5\dot{1}2$  as a fraction.  
Give your answer in its simplest form.

$$\begin{aligned}x &= 0.5121212\dots \\10x &= 5.121212\dots \\1000x &= 512.1212\dots \\990x &= 507\end{aligned}$$

$$\frac{507}{990}$$

$$\frac{169}{330}$$

(3)

7. Convert  $0.4515151\ldots$  to a fraction.  
Give your answer in its simplest form.

$$x = 0.4515151\ldots$$

$$10x = 4.515151\ldots$$

$$1000x = 451.5151\ldots$$

$$990x = 447$$

$$\frac{447}{990}$$

$$\frac{149}{330}$$

(3)

8. Write  $1.2\dot{4}$  as a mixed number.  
Give your answer in its simplest form.

$$x = 1.2444\ldots$$

$$10x = 12.444\ldots$$

$$100x = 124.444\ldots$$

$$90x = 112$$

$$x = \frac{112}{90}$$

$$\frac{56}{45} = 1\frac{11}{45}$$

$$1\frac{11}{45}$$

(3)

9. Write  $2.1\dot{6}\dot{5}$  as a mixed number.  
Give your answer in its simplest form.

$$x = 2.1656565\ldots$$

$$10x = 21.656565\ldots$$

$$1000x = 2165.656565\ldots$$

$$990x = 2144$$

$$\frac{2144}{990} = \frac{1072}{495}$$

$$2\frac{82}{495}$$

(3)

10. Write the numbers below in order.  
Start with the smallest.

$$\frac{11}{23}$$

$$0.4\dot{7}2$$

$$\frac{5}{11}$$

$$\frac{26}{55}$$

$$\frac{598}{1265}$$

$$\frac{25}{55}$$

$$\frac{575}{1265}$$

$$x = 0.4727272\ldots$$

$$10x = 4.72727\ldots$$

$$1000x = 472.727272\ldots$$

$$990x = 468$$

$$x = \frac{468}{990} = \frac{26}{55}$$

$$\frac{605}{1265}$$

$$\frac{5}{11}, 0.4\dot{7}2, \frac{11}{23}$$



## Topic 11: Iteration

MathsWatch Video: 180

1. The table below shows values of  $x$  and  $y$  for  $y = x^3 - 8x - 10$



$x$	0	1	2	3	4
$y$	-10	-17	-18	-7	22

Between which two consecutive integers is there a solution to the equation  $x^3 - 8x - 10 = 0$ ?

Explain your answer.

$x = \dots\dots\dots 3 \dots\dots\dots$  and  $x = \dots\dots\dots 4 \dots\dots\dots$

When  $x = 3$   $3^3 - 8 \times 3 - 10 = -7$

When  $x = 4$   $4^3 - 8 \times 4 - 10 = 22$

Since there is a change in sign, there will be a solution (2)

2. Using  $x_{n+1} = 8 - \frac{5}{x_n^2}$



with  $x_0 = 1$

find the values of  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$

$$x_1 = 8 - \frac{5}{1^2} = 3$$

$$x_2 = 8 - \frac{5}{3^2} = 7.444\dots$$

$$x_3 = 8 - \frac{5}{7.444\dots^2} = 7.909779461$$

$$x_4 = 8 - \frac{5}{7.9097\dots^2} = 7.920082617$$

$x_1 = \dots\dots\dots 3 \dots\dots\dots$

$x_2 = \dots\dots\dots 7.4 \dots\dots\dots$

$x_3 = \dots\dots\dots 7.909779461 \dots\dots\dots$

$x_4 = \dots\dots\dots 7.920082617 \dots\dots\dots$

(4)

8. (a) Show that the equation  $20 - x^3 - 7x^2 = 0$  can be rearranged to give

$$x = \frac{20}{x^2} - 7 \quad 20 - 7x^2 = x^3$$

$$\frac{20}{x^2} - 7 = x$$

$$x = \frac{20}{x^2} - 7$$

(2)

(b) Using  $x_{n+1} = \frac{20}{x_n^2} - 7$  with  $x_0 = -9$

find the values of  $x_1$ ,  $x_2$  and  $x_3$

$$x_1 = \frac{20}{(-9)^2} - 7 = -6.75308642$$

$$x_2 = \frac{20}{(-6.75\ldots)^2} - 7 = -6.561643673$$

$$x_3 = \frac{20}{(-6.56\ldots)^2} - 7 = -6.535451368$$
$$x_1 = -6.75308642$$
$$x_2 = -6.561643673$$
$$x_3 = -6.535451368$$

(3)

(b) Explain what the values of  $x_1$ ,  $x_2$  and  $x_3$  represent

$x_1$ ,  $x_2$  and  $x_3$  are increasingly accurate estimates to a solution of  $20 - x^3 - 7x^2 = 0$

(2)

10. (a) Show that the equation  $x^4 - 5x + 1 = 0$  has a root between  $x = 1.5$  and  $x = 2$

when  $x = 1.5$

$$1.5^4 - 5 \times 1.5 + 1 = -1.4375$$

when  $x = 2$

$$2^4 - 5 \times 2 + 1 = 7$$

Since there is a change of sign  
 $x^4 - 5x + 1 = 0$  has a solution between  $x = 1.5$  and  $x = 2$  (2)

- (b) Use the iteration formula  $x_{n+1} = \sqrt[3]{5 - \frac{1}{x_n}}$  three times with  $x_0 = 1.5$

to find an estimate for the solution of  $x^4 - 5x + 1 = 0$

$$x_1 = \sqrt[3]{5 - \frac{1}{1.5}} = 1.630324416$$

$$x_2 = \sqrt[3]{5 - \frac{1}{1.630..}} = 1.636980507$$

$$x_3 = \sqrt[3]{5 - \frac{1}{1.6369..}} = 1.637290685$$

11. The equation  $x^3 - 2x^2 + 19 = 0$  has a root in the interval  $(-3, -2)$



Use an appropriate iteration formula to find an approximate to 2 decimal places for the root of  $x^3 - 2x^2 + 19 = 0$  in the interval  $(-3, -2)$

$$x^3 = 2x^2 - 19$$

$$x = \sqrt[3]{2x^2 - 19}$$

$$x_{n+1} = \sqrt[3]{2(x_n)^2 - 19}$$

$$x_0 = -2$$

$$x_1 = -2.223980091$$

$$x_2 = -2.08835773$$

$$x_3 = -2.174183353$$

$$x_4 = -2.121313821$$

$$x_5 = -2.154438665$$

$$x_6 = -2.133900886$$

$$x_7 = -2.146718196$$

$$x_8 = -2.138751563$$

$$x_9 = -2.143715813$$

$$x_{10} = -2.140627311$$

$$\begin{array}{r} -2.14 \\ \hline \end{array}$$

(5)



## Topic 12: Completing the Square

MathsWatch Videos: 209

Georgina rewrites the expression  $x^2 + px + q$  by completing the square.  
He correctly obtains  $(x - 5)^2 + 31$

Work out the values of  $p$  and  $q$ .

$$\begin{aligned}(x-5)^2 + 31 \\(x-5)(x-5) + 31 \\x^2 - 10x + 25 + 31 \\x^2 - 10x + 56\end{aligned}$$

$$p = -10 \text{ and } q = 56$$

(3)

Write  $x^2 - 3x + 7$  in the form  $(x + a)^2 + b$

$$\begin{aligned}\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 7 \\ \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{28}{4} \\ \left(x - \frac{3}{2}\right)^2 + \frac{19}{4}\end{aligned}$$

Express  $3x^2 + 18x - 1$  in the form  $a(x + b)^2 + c$

$$3\left(x^2 + 6x - \frac{1}{3}\right)$$

$$3\left[(x+3)^2 - 9 - \frac{1}{3}\right]$$

$$3\left[(x+3)^2 - \frac{27}{3} - \frac{1}{3}\right]$$

$$3\left[(x+3)^2 - \frac{28}{3}\right]$$

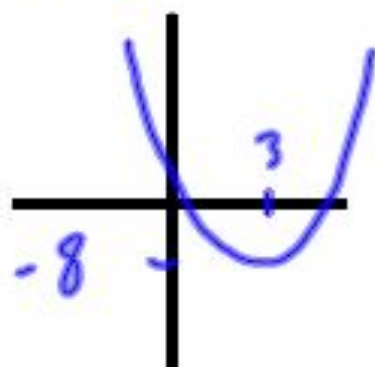
$$3(x+3)^2 - 28$$

.....  
(3)

Use completing the square to find the minimum point of the curve  $y = x^2 - 6x + 1$

$$y = (x-3)^2 - 9 + 1$$

$$y = (x-3)^2 - 8$$



.....  
(3, -8)

(4)

Use completing the square to find the minimum point of the curve  $y = x^2 + 4x + 7$

$$y = (x+2)^2 - 4 + 7$$

$$y = (x+2)^2 + 3$$

$$(-2, 3)$$